: Cylinder :

* **Eqⁿ of cylinder :**

To find the eqⁿ to the cylinder whose generators are ||el to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ & intersect the curve
$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ & $z = 0$
Let $P(x_1, y_1, z_1)$ be any point on generator cylinder
Then eqⁿ of the generator through P parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$, is

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

Q. Find the eqⁿ of the cylinder whose guiding curve is $ax^2 + by^2 = 2z$ & $lx + my + nz = p$ & generators parallel to the z-axis

⇒ Since the generators are parallel to the z-axis we have to eliminate z from the above two eqⁿ of guiding curve.

$$ax^2 + by^2 = 2z$$

$$\Rightarrow z = \frac{ax^2 + by^2}{2}$$

& $lx + my + nz = p$

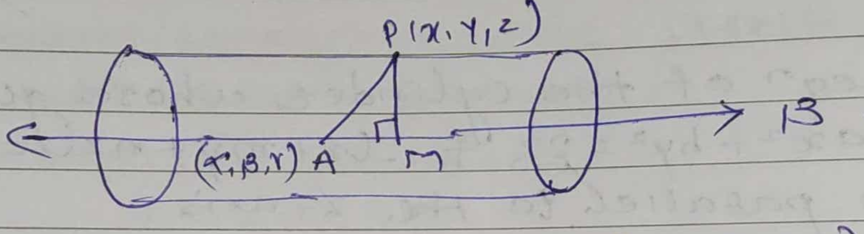
$$\Rightarrow lx + my + n\left(\frac{ax^2 + by^2}{2}\right) = p.$$

∴ required eqⁿ is,

$$n(ax^2 + by^2) + 2lx + 2my - 2p = 0$$

* Right circular cylinder :
is a surface generated by st. line parallel to a fixed line & is at a const dist. from it.

* Eqⁿ of a Right circular cylinder :
• General form:
eqⁿ of right circular cylinder whose radius r & axis is the line
$$\frac{x-\alpha}{l} = \frac{x-\beta}{m} = \frac{z-\gamma}{n}$$



Let A be the point (α, β, γ) & AB the axis whose eqⁿ are
$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}.$$
dr's of line are l, m, n.
∴ dc's are
$$\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$$

~~P be~~ $P(x, y, z)$ be any point on cylinder & $PM \perp$ to AB
$PM = r =$ radius.
$$PA = \sqrt{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}$$

AM = proj. of AP on line AB.
$$AM = \frac{(x-\alpha)l}{\sqrt{l^2+m^2+n^2}} + \frac{(y-\beta)m}{\sqrt{l^2+m^2+n^2}} + \frac{(z-\gamma)n}{\sqrt{l^2+m^2+n^2}}$$

$\therefore$ From ΔAPM

$$AP^2 = AM^2 + PM^2$$

i.e. $(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 = r^2 + \dfrac{[l(x-\alpha)+m(y-\beta)+n(z-\gamma)]^2}{l^2+m^2+n^2}$ --- (1)

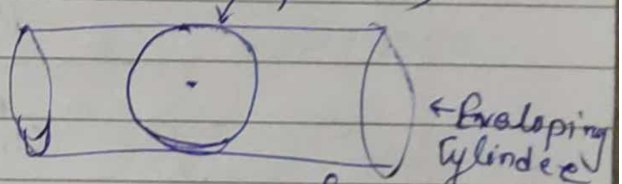
is required eqn of cylinder.

* Eveloping Cylinder :-

Defn :-

The locus of the tangent lines to a given surface & llel to a given line is a cylinder & is called enveloping of the surface

- Eqn of Envoloping cylinder :-



if eqn of sphere is $x^2+y^2+z^2 = a^2$, whose cylinder's generators are llel to line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

Let $p(x_1, y_1, z_1)$ be any pt. on tangent to sphere $x^2+y^2+z^2 = a^2$ & llel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$

then eqn of tangent is

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} = r \quad \text{— (1)}$$

$\therefore$ From (1)

$x = x_1 + lr, \quad y = y_1 + mr, \quad z = z_1 + nr$

put in $x^2+y^2+z^2 = a^2$

$\therefore (x_1+lr)^2 + (y_1+mr)^2 + (z_1+mr)^2 = a^2$

$\therefore (l^2+m^2+n^2)r^2 + 2r(lx_1 + my_1 + nz_1) + x_1^2+y_1^2+z_1^2 - a^2 = 0.$

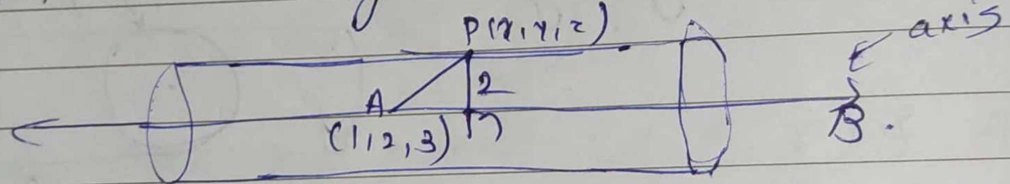
line (1) tangent to sphere in only one point

* Required things.
1. One point on axis
2. dr's of axis
3. radius of cylinder

Q. Find the eq^n of right circular cylinder of radius 2 & eq^n of axis is

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}$$

$\Rightarrow$ Given
1. point on axis = (1, 2, 3)
2. dr's of axis = 2, -3, 6
3. Radius of cylinder = 2.



Let $P(x, y, z)$ be any point on cylinder & $A(1,2,3)$ be one point on axis & M be the $\perp^{ar}$ from P on axis.

$\therefore$ by dist. formula.

$$AP = \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$$

Dist. PM = radius = 2

& dist AM = proj. of AP on Axis.

$\therefore$ from dr's (2, -3, 6)

dc's of axis are

$$\left(\frac{2}{\sqrt{2^2+(-3)^2+6^2}}, \frac{-3}{\sqrt{2^2+(-3)^2+6^2}}, \frac{6}{\sqrt{2^2+(-3)^2+6^2}}\right)$$

$$\left(\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}\right)$$

$\therefore AM = \frac{2}{3}(x-1) - \frac{3}{7}(y-2) + \frac{6}{7}(z-3)$

$AM = \frac{2x - 3y + 6z - 14}{7}$

$\therefore$ from ΔAPM

$AP^2 = AM^2 + MP^2$

$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = \frac{(2x - 3y + 6z - 14)^2}{49} + 4$

simplify.

$\Rightarrow 45x^2 + 40y^2 + 13z^2 + 12xy + 36yz - 24xz - 42x - 280y - 126z + 294 = 0.$

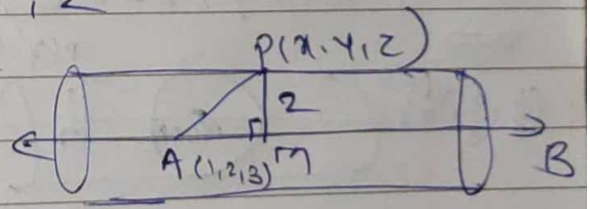
which is required eq^n of cylinder.

Q. Find the eq^n of right circular cylinder of radius 2, whose axis passes through $(1,2,3)$ & has dc's proportion to $(2,1,2)$.

$\Rightarrow$ ① one point on axis $= (1,2,3)$

② drs of axis are $2, 1, 2$

③ Radius of cylinder $= 2$



$\therefore AP = \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$

$= \sqrt{\quad " \quad}$

& $PM = 2$

$AM =$ proj. of AP on Axis AB.

dr's of axis are $2, 1, 2$

$\therefore$ dcs are $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

$\therefore AM = \frac{2}{3}(x-1) + \frac{1}{3}(y-2) + \frac{2}{3}(z-3).$

$$AM = \frac{2x+y+2z-10}{3}$$

from ΔAPM.

$$AP^2 = AM^2 + MP^2$$

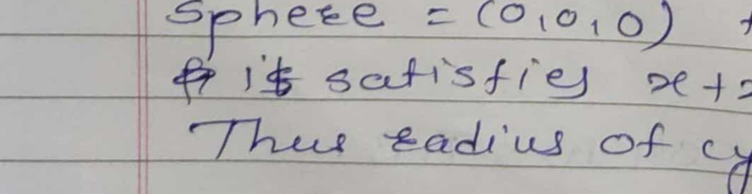
$$\therefore (x-1)^2 + (y-2)^2 + (z-3)^2 = \frac{1}{9}[2x+y+2z-10]^2 + 4$$

$$\Rightarrow 9(x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9]$$
$$= 4x^2 + y^2 + 4z^2 + 100 + 4xy + 8xz - 40x + 4yz - 20y - 40z + 4.$$

$$\therefore 5x^2 + 8y^2 + 5z^2 + 22x - 16y - 14z - 4xy - 8xz - 4yz - 23 = 0.$$

Q. Find the eqn of the right circular cylinder which passes through the section of the sphere $x^2+y^2+z^2=25$, $x+2y+2z=0$.

$\Rightarrow$



Given sphere $x^2+y^2+z^2=25$
$\therefore$ radius of sphere $=5$ & centre of sphere $=(0,0,0)$ this centre is satisfies $x+2y+2z=0$
Thus radius of cylinder = radius of sphere $=5$
i.e the required cylinder is the enveloping cylinder of given sphere
Now we know that

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① (0,0,0) point on axis
② (1,2,2) dr's of axis ($\because$ the axis is normal to the plane).
③ radius of cylinder = 5

Let $P(x,y,z)$ be any point on the cylinder & $A(0,0,0)$ is one point on axis

Let M be the foot of $\perp^{ar}$ from point P on the axis

$$\therefore AP = \sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$$

$PM = 5$ (radius of cylinder)

AM = proj. of AP on axis.

Now $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ are dc's of axis

$$\therefore AM = \frac{1}{3}(x-0) + \frac{2}{3}(y-0) + \frac{2}{3}(z-0)$$

$$= \frac{x+2y+2z}{3}$$

& From ΔAPM

$$AP^2 = AM^2 + PM^2$$

$$x^2+y^2+z^2 = \frac{(x+2y+2z)^2}{9} + 25$$

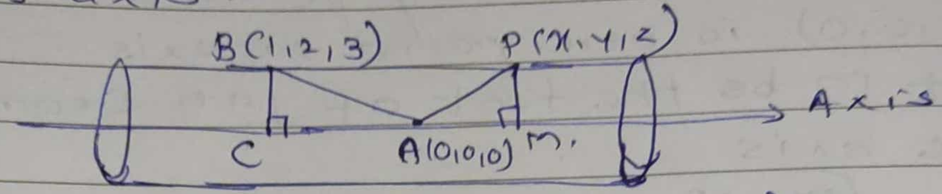
simplify.

$$9(x^2+y^2+z^2) = \frac{(x+2y+2z)^2}{\cancel{9}} + 25\times 9$$

$$\therefore 8x^2+5y^2+5z^2-4xy-4zx-8yz-225=0.$$

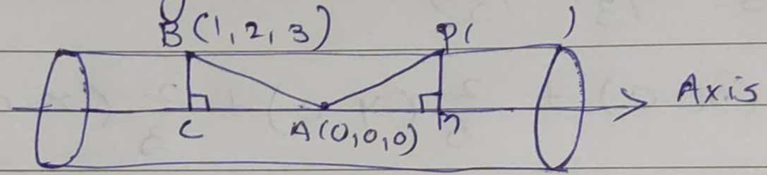
Q. Determine the radius of the right circular cylinder & hence obtain its eqn if the cylinder passes through the point (1,2,3) & has $\frac{x}{3} = \frac{y}{2} = \frac{z}{1}$ as axis.

$\Rightarrow$



(i) one point on axis A (0,0,0).
(ii) Dr's of axis are 3, 2, 1
∴ To find Radius of cylinder
(iii) Radius of cylinder = ?

$\Rightarrow$



~~We have~~
Let $\angle CAB = \theta$.

then $\cos\theta = \frac{3\times1 + 2\times2 + 1\times3}{\sqrt{3^2+2^2+1^2}\sqrt{1^2+2^2+3^2}}$

$= \frac{3+4+3}{\sqrt{14}\sqrt{14}}$

$= \frac{10}{14} = \frac{5}{7}$

& $\sin\theta = \sqrt{1-\cos^2\theta}$.

$= \sqrt{1-\frac{25}{49}}$

$= \frac{2\sqrt{6}}{7}$.

$\therefore (BC)^2 = r^2 = 14\sin^2\theta$

$= \frac{14\times24}{49}$

$= \frac{48}{7}$.

consider the following steps.

Step-I : Let $P(x, y, z)$ be any point on generator of RCC. Let M be $\perp$ from P on axis.

$$(AP) = \sqrt{x^2 + y^2 + z^2}$$

$$PM = \sqrt{\frac{48}{7}}$$

dc's of axis are $\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}$.

AM = proj. of AP on axis

$$= \frac{3}{\sqrt{14}}x + \frac{2}{\sqrt{14}}y + \frac{z}{\sqrt{14}}$$

$$= \frac{3x + 2y + z}{\sqrt{14}}$$

From ΔAMP.

$$(AP)^2 = (AM)^2 + (PM)^2$$

$$\Rightarrow x^2 + y^2 + z^2 = \frac{(3x + 2y + z)^2}{14} + \frac{48}{7}.$$

$$14(x^2 + y^2 + z^2) = 9x^2 + 4y^2 + z^2 + 12xy + 6xz + 4yz + 96.$$

$$\Rightarrow 5x^2 + 10y^2 + 13z^2 - 12xy - 6xz - 4yz - 96 = 0.$$

which is required eqn of RCC.